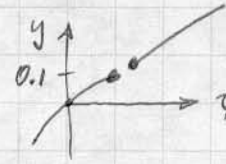
OE Fluids F14 Solution Fall '07

a) It's easiest to start with <u>middle piece</u>

$\frac{\partial \psi}{\partial y} = u = 1 + 5y \rightarrow \psi_{mid}(y) = y + \frac{5}{2}y^2 + \phi$ can set $\phi = 0$ (no effect on velocity)

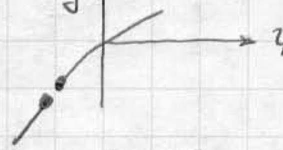
<u>Top piece</u>: $\frac{\partial \psi}{\partial y} = u = 1.5 \rightarrow \psi_{top}(y) = 1.5y + \phi_{top}$



Require continuity $\psi_{top} = \psi_{mid}$ at $y = 0.1$

$$1.5 \cdot (0.1) + \phi_{top} = 0.1 + \frac{5}{2}(0.1)^2 \rightarrow \phi_{top} = -0.025$$

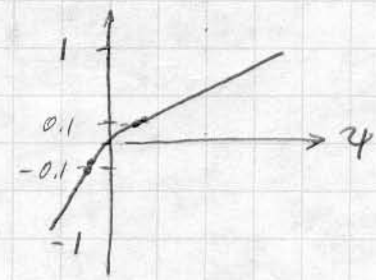
<u>Bottom piece</u> $\frac{\partial \psi}{\partial y} = u = 0.5 \rightarrow \psi_{bot}(y) = 0.5y + \phi_{bot}$



Require continuity $\psi_{bot} = \psi_{mid}$ at $y = -0.1$

$$0.5(-0.1) + \phi_{bot} = -0.1 + \frac{5}{2}(-0.1)^2 \rightarrow \phi_{bot} = -0.025$$

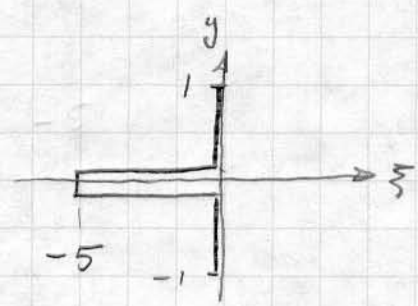
$$\psi(y) = \begin{cases} 1.5y - 0.025, & 0.1 < y < 1 \\ y + 2.5y^2, & -0.1 < y < 0.1 \\ 0.5 - 0.025, & -1 < y < -0.1 \end{cases}$$



b) $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\frac{\partial u}{\partial y}$

Evaluate for each piece:

$$\xi(y) = \begin{cases} 0, & 0.1 < y < 1 \\ -5 & -0.1 < y < 0.1 \\ 0 & -1 < y < 0.1 \end{cases}$$



c) Helmholtz Eq'n: $\frac{D\xi}{Dt} = 0$ or $\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} = 0$

For this case, $\frac{\partial \xi}{\partial t} = 0$ (steady) and $v = 0$,

so Helmholtz becomes $u\frac{d\xi}{dx} = 0$

But for ξ in this case, $\frac{d\xi}{dx} = 0$, so $\frac{D\xi}{Dt} \stackrel{\checkmark}{=} 0$ valid.